Beyond Euclid: An Educational VR Journey into Spherical Geometry

Agapi Chrysanthakopoulou   Theofilos Chrysikopoulos   Leandros-Nikolaos Arvanitopoulos
Konstantinos Moustakas[*†]

Department of Electrical and Computer Engineering, University of Patras

Abstract

This paper introduces an innovative educational virtual reality (VR) experience aimed at immersing users in the complexities of spherical geometry. Focused on spherical triangles, navigation methods, and map projections, the VR journey offers an interactive platform for learning about Earth's unique geometry. Users engage in modules that explore the properties of spherical triangles, challenges in earth navigation, and the intricacies of map projections, providing them with insights into geodesics and their practical applications. An innovative calculator is designed that allows users to use a spaceship's control system for doing calculations, thus creating a more intuitive and engaging experience.

Index Terms: Human-centered computing—Virtual learning—Spherical geometry—Virtual reality—3D Graphics;

1 Introduction

Spherical geometry, a branch of geometry dedicated to the study of objects on the surface of a sphere, introduces a unique set of principles that diverge from the familiar Euclidean geometry of flat surfaces. Spherical trigonometry focuses on the relationships and properties of triangles formed on the surface of a sphere and maintains distinct characteristics. Its practical applications in various fields, including astronomy, geodesy, and navigation underscore its significance as an indispensable tool in interpreting and navigating the 3D aspects of our world. In geodesy, understanding the geometry of the Earth's surface is fundamental for accurate map projections and spatial measurements.

Virtual reality learning environments (VRLEs) have been proven to increase engagement, presence, and usability. Game-based VRLEs in various fields have also gained attention due to their combination of likability and usability [1, 2]. Utilizing 3D techniques in mathematical education is paramount for enhancing conceptual understanding. By providing a visual and interactive dimension to abstract concepts, 3D representations enable learners to grasp mathematical ideas more intuitively [3]. The spatial perspective stimulates a deeper comprehension of geometric principles, bridging the gap between theory and application.

Our proposed 3D user interface (UI) design aims to deliver an immersive and intriguing experience to the end-users, where they will learn about how spherical geometry works and its basic concepts and grasp its importance in navigation and map creation. The aforementioned interactive UI presents a gamified scenario, where the player uses a spaceship's controls to do calculations and travel through space. The 3D controls with haptic feedback give the user a more engaging and intuitive role unlike classic calculators, providing better learning outcomes after completing the tasks.

[*]e-mail: moustakas@upatras.gr
[†]URL: www.vvr.ece.upatras.gr

2 Design

To ensure our solution was suitable for the contest, we considered portability, hardware, and spatial limitations. We also focused on creating a user-friendly design and gameplay that would be interesting and easy for players to learn and interact with.

First, we want to introduce the user to Spherical Geometry (SG) by comparing it with Euclidean Geometry(EG) and demonstrating some key differences that are briefly shown in Table 1.

Euclidean Geometry	Spherical Geometry
Triangle angles add to 180°	180°<Angles sum<540°
Parallel lines exist	Any two lines intersect twice
Shortest path is a straight line	Shortest path is along an arc

Table 1: Euclidean vs Spherical Geometry

While exploring spherical trigonometry, the user can draw triangles on a 3D sphere and become familiar with how the angles are calculated and some fundamental formulas like the area of a triangle. They can then expand their knowledge by discovering how to find the area of any polygon on the sphere by triangulating it (Fig. 1).

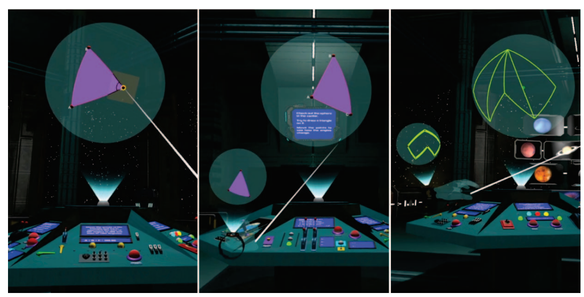

Figure 1: Exploring triangles and polygons on a sphere.

It is mathematically impossible to accurately represent a spherical object in a 2D plane, but projection mappings such as cylindrical, conic, and azimuthal, offer trade-offs between details and precision. The cylindrical are the projections of the sphere to a cylinder, which then unravels into a plane by introducing a cut. Intuitively, the points of the sphere closest to the curvature and the tangent of the shape it is projected onto, display less distortion than those further away, therefore cylindrical better describes the equator (Fig. 2).

In the game, the user becomes familiar with two cylindrical projections: the Equirectangular and Lambert's Cylindrical Equal-Area. The first represents meridians as equally spaced vertical lines and parallels as equally spaced horizontal lines, creating a rectangular grid. The second maintains accurate relative sizes of areas on the map and is useful for thematic mapping where maintaining the size ratio between two areas is crucial.

On a sphere, unlike the straight lines in Euclidean geometry, the shortest path between two points is the smaller arc of the great circle that passes through those two points. A great circle is a circle formed on the surface of the sphere where it intersects with a plane passing through its center. Great circle arcs are important in mapping, navigation, air travel, and other applications that require

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precise representation of the shortest path between two locations. The true shortest path on a spherical surface when it is projected onto a flat map, the result is a 2D arc that preserves the geodesic nature of the path. The equirectangular and Lambert projections are used in the application to project the shortest paths between two points on Earth and two points on the world map (Fig. 2).

A spherical triangle has three edges which are the shortest paths between its three points. A polygon on a sphere's surface can be calculated by determining the areas of the triangles that make it up. The ear-clipping method is employed for triangulating the polygon, a widely used and simple technique. This method examines each point of the polygon, identifying if it forms an "ear," and subsequently removes it from the polygon. This process iterates until only three points remain, resulting in a triangulated representation of the original polygon.

Figure 2: The station for calculating the shortest path, triangles and polygons on Earth and its projections.

Calculator and Interactions Throughout the game, the player using the controllers can do various calculations and move around freely in the spaceship. Unlike a classic calculator, the interface is a spaceship's control room with buttons, levers, sliders, etc. Each element is assigned with a distinct operation (e.g. a lever is for: $\cdot R^2$) so the user isn't consumed with individual operations (e.g. multiply, square) but rather with a better understanding of the formula itself. The interactions are fun and straightforward, grabbing and pushing the elements naturally, and every interaction provides haptic feedback for a more immersed feeling. Also, by utilizing the World-In-Miniature and Voodoo Dolls techniques the user can work on a handheld representation of the sphere with a virtual pen and have better control of certain manipulations. Long-distance interactions are used to eliminate real-world limitations and give the player additional possibilities.

3 Game Logic

The present work was designed in the Unity3D engine utilizing the OpenXR plug-in framework to support cross-platform experiences. Through the development process, the Oculus Quest head-mount display was used.

First, the player finds themself in a spaceship bridge where they see an intro text on a monitor and some tutorial info. The premise is that the player is on a spacecraft lost in space and their goal is to return to Earth by completing a series of tasks. When they become more familiar with the controls they can press a button to open the door to enter the main room.

In the main room (Figure 3), there are various monitors and control panels. A large screen is always there to provide hints and help as the player progresses. In the center of the room, there is a large sphere hologram. There are 4 distinct stations that the player must visit to complete their mission.

Euclidean to Spherical A simple task where the player has to turn a wheel in order to observe the key differences between Euclidean and spherical geometry and how the relations change from the plane to the sphere.

Spherical Trigonometry The station in the middle of the room focuses on some spherical trigonometry's basic math. The player has

Figure 3: The main room.

to do 3 tasks. First, players draw a triangle on a sphere, strategically adjusting its points to ensure the sum of angles reaches 180° and later, 540°. Then they are challenged to choose a triangle on the sphere and determine its area by using the interactive 3D handles available on the control panel. Finally, they are called to draw a polygon on the sphere and employ the triangulation method to find its area.

Map projections and Shortest Path Here the player can explore the world of cartography as they draw lines, triangles, or polygons on the Earth's surface with the virtual pen. The paths are projected onto two distinct mapping systems: the Equirectangular and Lambert Cylindrical Equal-Area projections. The player can calculate the shortest path between two points on Earth or calculate the area of a country by drawing a polygon around its borders as detailed as they want. Apart from the visual results they can read about the mathematical backgrounds of these transformations to understand why the deformations on the maps occur.

The Quiz The player can now utilize all the information they have gathered to answer three questions that will lead them to their final destination. The quiz is on a tablet that the player can take with them while they revisit the previous stations to solve the problems. Even if they do not want to perform the calculations, by now, they have acquired the necessary knowledge of spherical geometry to answer the questions intuitively.

Finally, the user can see that with each task completed, they are getting closer to Earth with their spaceship but can continue to use and experiment with all the stations for a deeper understanding of spherical geometry.

4 Conclusion

The present work discusses the design and development of an educational game with various tasks, that focus on the users' education on spherical geometry and its main applications. Apart from the gamification elements, an innovative 3D calculator was implemented, to offer the users a more engaging experience and an intuitive understanding of the math behind known real-world problems.

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